Diploma Programme

Mathematics mind map

Analysis and approaches AHL

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Mathematics mind map

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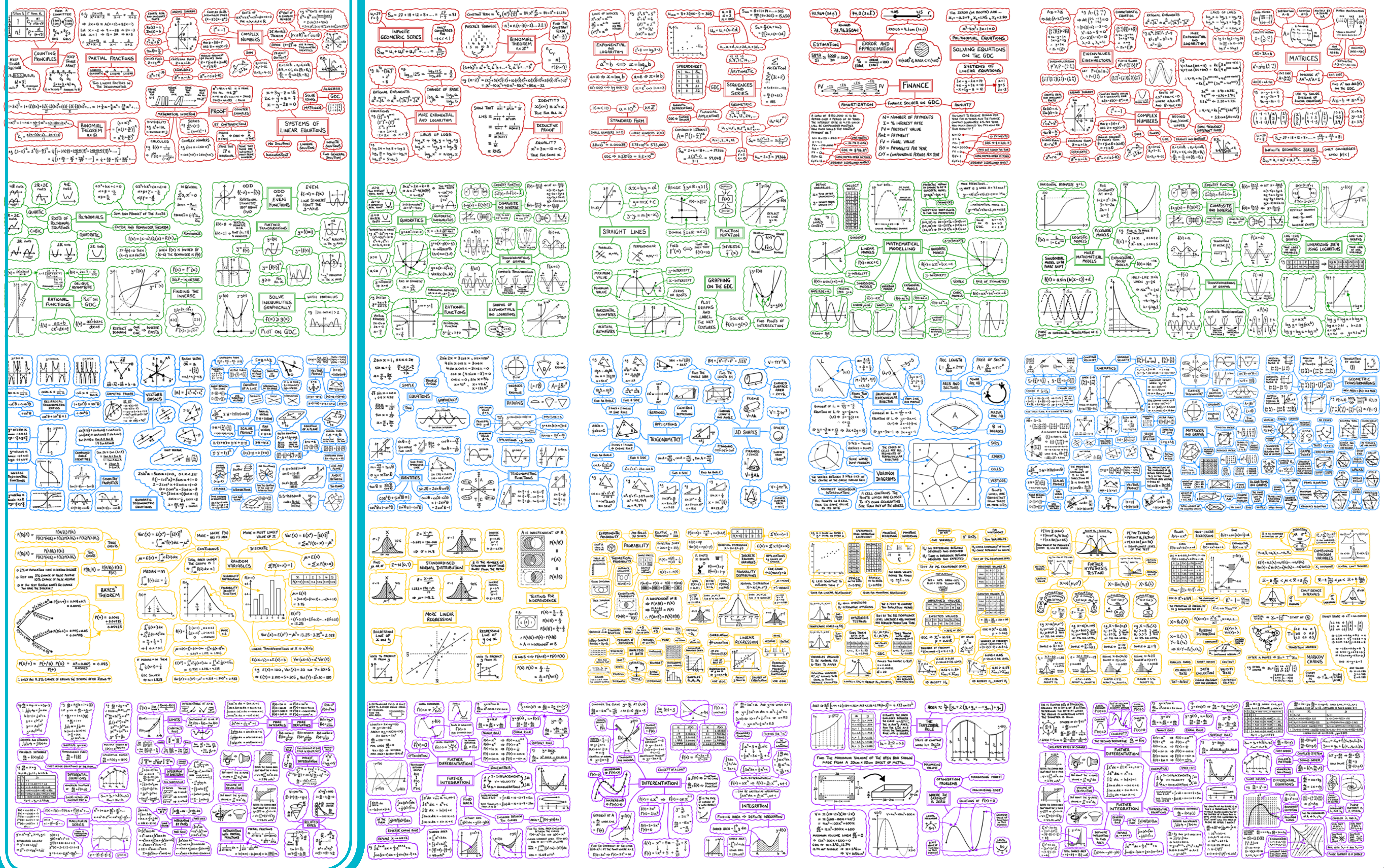
	Analysis and approaches AHL	Analysis and approaches SL	Common content	Applications and interpretation SL	Applications and interpretation AHL
Number and algebra					
Functions					
Geometry and trigonometry					
Statistics and probability					
Calculus					



Number and algebra

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COUNTING PRINCIPLES

	n FROM n	r FROM n
COMB	1	$^nC_r = \frac{n!}{r!(n-r)!}$
PERM	$n!$	$^nP_r = \frac{n!}{(n-r)!}$

FORM A COMMITTEE

8 BOYS 5 GIRLS → 4 BOYS 3 GIRLS

$^8C_4 \times {}^5C_3$ (BOYS, GIRLS)

KEEP THINGS TOGETHER

$B_1B_2\boxed{G_1G_2G_3}B_3B_4B_5$

$6! \times 3!$ (6 UNITS; REARRANGE THE G's)

KEEP THINGS APART

$\_B_1\_B_2\_B_3\_B_4\_$ (G_1, G_3, G_2)

$5\times4\times3\times4!$ (G's IN THE GAPS; REARRANGE THE B's)

PARTIAL FRACTIONS

e.g. $\frac{2x+13}{x^2+x-2} \equiv \frac{A}{x-1} + \frac{B}{x+2}$

$\Rightarrow 2x+13 \equiv A(x+2) + B(x-1)$

Let $x=-2 \Rightarrow 9=-3B \Rightarrow B=-3$

Let $x=1 \Rightarrow 15=3A \Rightarrow A=5$

$\therefore \frac{2x+13}{x^2+x-2} \equiv \frac{5}{x-1} - \frac{3}{x+2}$

$\frac{\text{LINEAR}}{\text{QUADRATIC}} \Rightarrow \frac{A}{\text{LINEAR}} + \frac{B}{\text{LINEAR}}$

TWO LINEAR FACTORS IN THE DENOMINATOR

COMPLEX NUMBERS

EQUATE REAL AND IMAGINARY PARTS

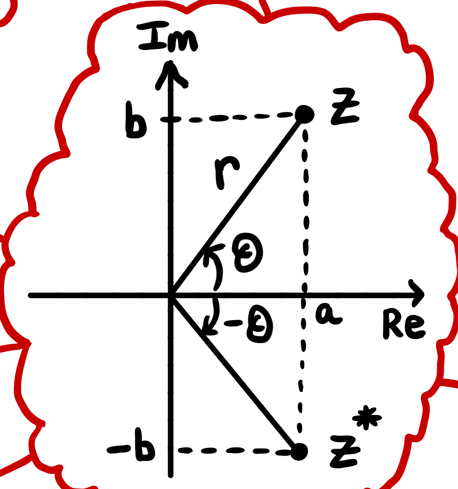
$\text{Re}(z)=a$

$\text{Im}(z)=b$

$r^2=a^2+b^2$

$\tan\theta = \frac{b}{a}$

ARGAND DIAGRAM



EULER'S FORM $z=re^{i\theta}$

CARTESIAN FORM $z=a+ib$

MODULUS-ARGUMENT (OR POLAR) FORM $z=r(\cos\theta+i\sin\theta) = r\,\text{cis}\,\theta$

CONJUGATE

$z^*=re^{-i\theta}$

$z^*=a-ib$

$z^*=r\,\text{cis}(-\theta)$

MOD $z=|z|=r$

ARG $z=\arg(z)=\theta$

$i^2=-1$

COMPLEX ROOTS IN CONJUGATE PAIRS $(x-z)(x-z^*)$

ROOTS OF $ax^4+bx^3+cx^2+dx+e=0$ FOR $a,b,c,d,e \in \mathbb{R}$

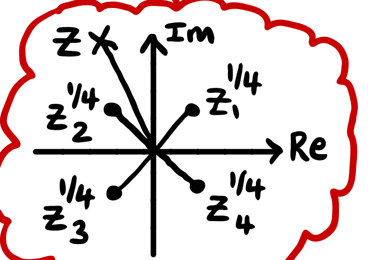
n^{th} ROOT OF A COMPLEX NUMBER

e.g 4^{TH} ROOTS OF $16\,\text{cis}\,100°$

$z^{1/4} = 16^{1/4}\,\text{cis}\left(\frac{100+360k}{4}\right)$

$= 2\,\text{cis}(25+90k)$

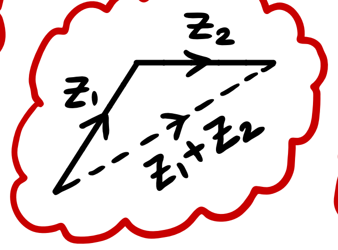
$= 2\,\text{cis}\,25°, 2\,\text{cis}\,115°, 2\,\text{cis}\,205°, 2\,\text{cis}\,295°$



DE MOIVRE'S THEOREM $(r\,\text{cis}\,\theta)^n \equiv r^n\,\text{cis}\,n\theta$

EXPAND $\left(z+\frac{1}{z}\right)^n \Rightarrow$ USE TO PROVE TRIGONOMETRIC IDENTITIES

SUM



$z_1=3+2i$

$z_2=1-5i$

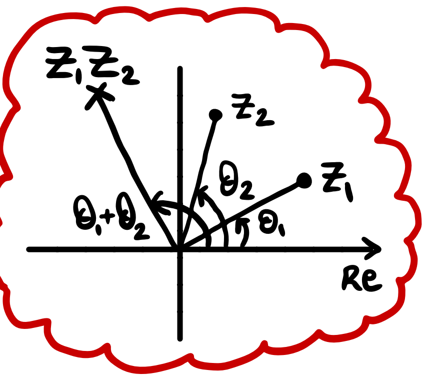
$z_1+z_2=4-3i$

PRODUCT OR QUOTIENT

$z_1=r_1\,\text{cis}\,\theta_1$, $z_2=r_2\,\text{cis}\,\theta_2$

$z_1z_2 = r_1r_2\,\text{cis}(\theta_1+\theta_2)$

$\frac{z_1}{z_2} = \frac{r_1}{r_2}\,\text{cis}(\theta_1-\theta_2)$



BINOMIAL THEOREM $n \in \mathbb{Q}$

$(1+3x)^{\frac{1}{2}} = 1+\left(\tfrac{1}{2}\right)(3x)+\frac{\left(\frac{1}{2}\right)\left(-\frac{1}{2}\right)}{2!}(3x)^2+\frac{\left(\frac{1}{2}\right)\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)}{3!}(3x)^3+\ldots = 1+\frac{3}{2}x-\frac{9}{8}x^2+\frac{27}{16}x^3+\ldots$

$(1+x)^n = 1+nx+\frac{n(n-1)}{2!}x^2+\frac{n(n-1)(n-2)}{3!}x^3+\ldots$

$^nC_r = \frac{n(n-1)(n-2)\ldots(n-r+1)}{r!}$

$(a+x)^n = \left[a\left(1+\frac{x}{a}\right)\right]^n = a^n\left(1+\frac{x}{a}\right)^n$

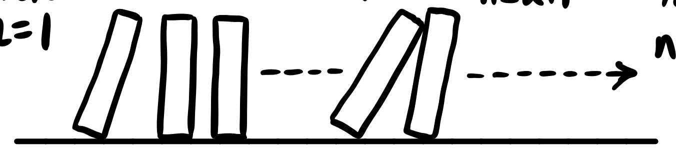
e.g $(3-x)^{-2} = 3^{-2}\left(1-\frac{x}{3}\right)^{-2} = \frac{1}{9}\left[1+(-2)\left(\frac{-x}{3}\right)+\frac{(-2)(-3)}{2!}\left(\frac{-x}{3}\right)^2+\frac{(-2)(-3)(-4)}{3!}\left(\frac{-x}{3}\right)^3+\ldots\right]$

$= \frac{1}{9}\left[1+\frac{2x}{3}-\frac{x^2}{3}+\frac{4x^3}{27}+\ldots\right] = \frac{1}{9}+\frac{2x}{27}-\frac{x^2}{27}+\frac{4x^3}{243}+\ldots$

PROOF

MATHEMATICAL INDUCTION

PROVE TRUE FOR $n=1$ — ASSUME TRUE FOR $n=k$ — PROVE TRUE FOR $n=k+1$ — HENCE TRUE FOR ALL $n \in \mathbb{Z}^+$



DIVISIBILITY: e.g n^3+11n IS DIVISIBLE BY 3

SERIES: e.g $\sum_{r=1}^{n} r(r!) = (n+1)!-1$

CALCULUS: e.g $f(x)=\frac{1}{1-x} \Rightarrow f^{(n)}(x) = \frac{n!}{(1-x)^{n+1}}$

COMPLEX NUMBERS: e.g $(\cos x+i\sin x)^n = \cos(nx)+i\sin(nx)$

COUNTER EXAMPLES

$n^2+41n+41$ IS A PRIME FOR ALL $n \in \mathbb{Z}^+$

$n=41 \Rightarrow n^2+41n+41 = 3403$

$3403 = 41\times83 \quad \therefore$ FALSE

BY CONTRADICTION

ASSUME A IS FALSE $\Rightarrow$ ERROR $\Rightarrow$ SO A IS TRUE

PROVE THAT $\sqrt{3}$ IS IRRATIONAL

PROVE THAT THERE ARE INFINITELY MANY PRIME NUMBERS.

SYSTEMS OF LINEAR EQUATIONS

$x+3y-z=15$

$2x+y+z=7$

$x-y-2z=0$

SOLVE USING... ALGEBRA / GDC / MATRICES

$\begin{pmatrix} 1 & 3 & -1 \\ 2 & 1 & 1 \\ 1 & -1 & -2 \end{pmatrix}\begin{pmatrix} 15 \\ 7 \\ 0 \end{pmatrix}$

NO SOLUTION — INCONSISTENT

UNIQUE SOLUTION

INFINITE SOLUTIONS — FIND GENERAL SOLUTION

Analysis and approaches AHL

Functions

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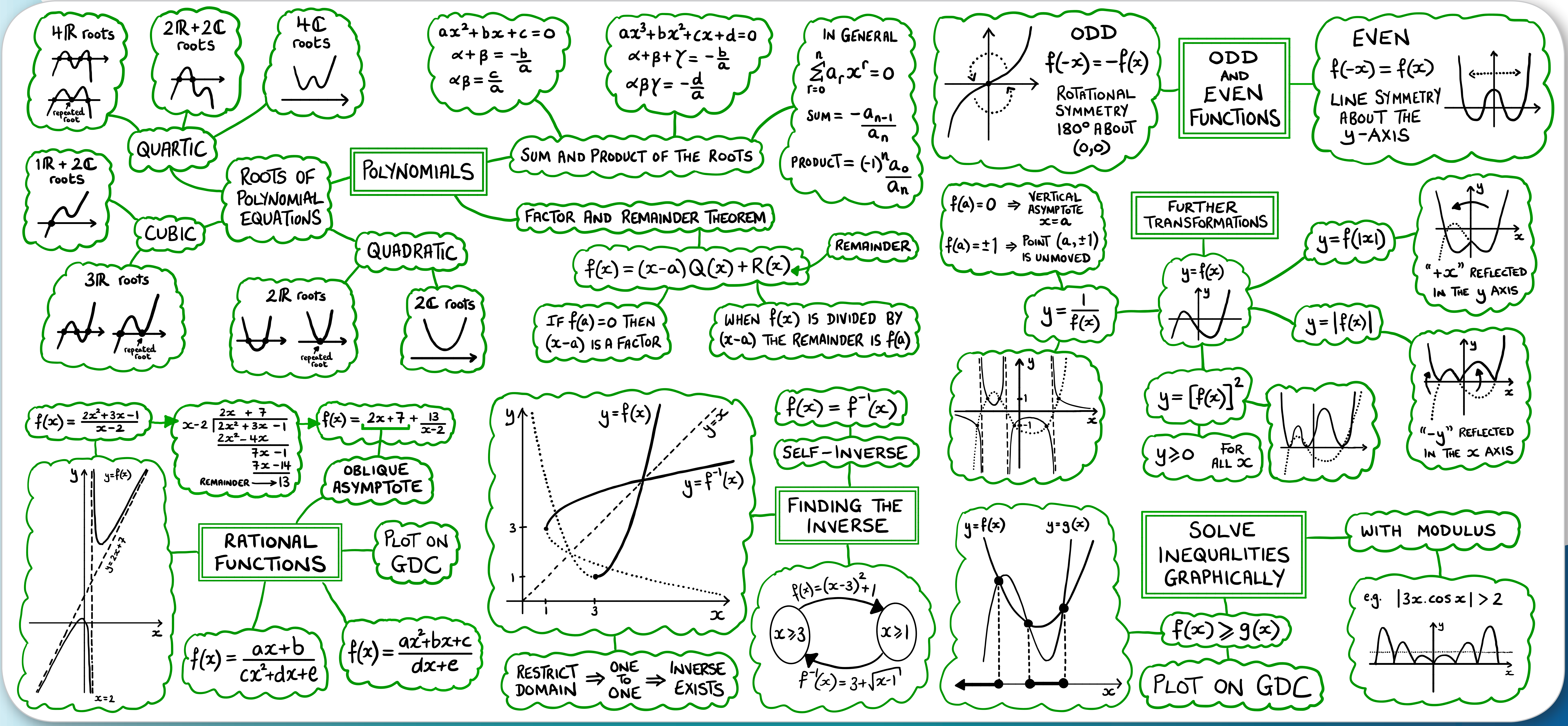

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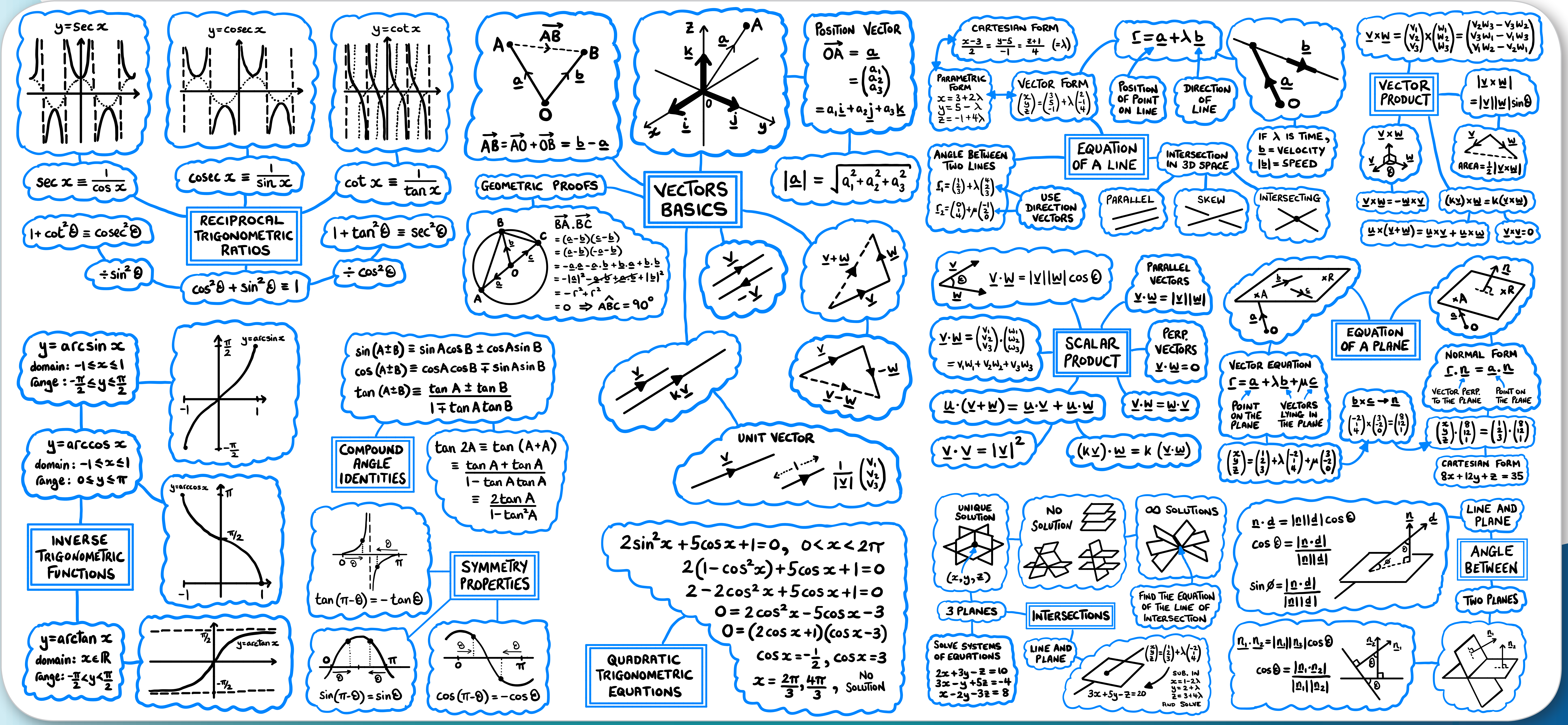
Geometry and trigonometry



Statistics and probability

BAYES' THEOREM

THREE EVENTS

$$P(B_i|A) = \frac{P(A|B_i)P(B_i)}{P(B_1)P(A|B_1)+P(B_2)P(A|B_2)+P(B_3)P(A|B_3)}$$

TWO EVENTS

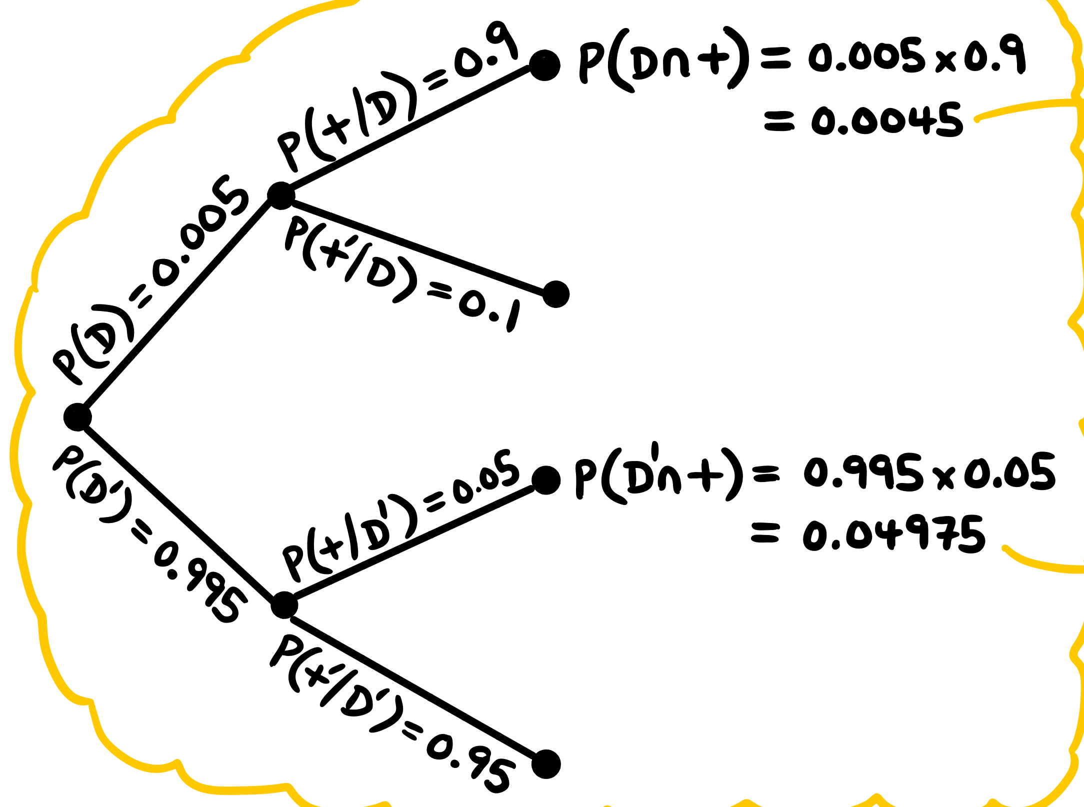
$$P(B_i|A) = \frac{P(A|B_i)P(B_i)}{P(B_1)P(A|B_1)+P(B_2)P(A|B_2)}$$

$$P(B_i|A) = \frac{P(A|B_i)P(B_i)}{P(A)}$$

⇒ $\frac{1}{2}$% OF POPULATION HAVE A CERTAIN DISEASE

⇒ TEST HAS 5% CHANCE OF FALSE POSITIVE, 10% CHANCE OF FALSE NEGATIVE

⇒ IF YOU TEST POSITIVE WHAT'S THE CHANCE YOU HAVE THE DISEASE?



$$P(+) = 0.0045 + 0.04975 = 0.05425$$

$$P(D/+) = \frac{P(+/D).P(D)}{P(+)} = \frac{0.9 \times 0.005}{0.05425} = 0.083$$

∴ ONLY AN 8.3% CHANCE OF HAVING THE DISEASE AFTER TESTING +

RANDOM VARIABLES

CONTINUOUS

$$Var(X) = E(X^2) - [E(X)]^2 = \int_{-\infty}^{\infty} x^2 f(x)\,dx - \mu^2$$

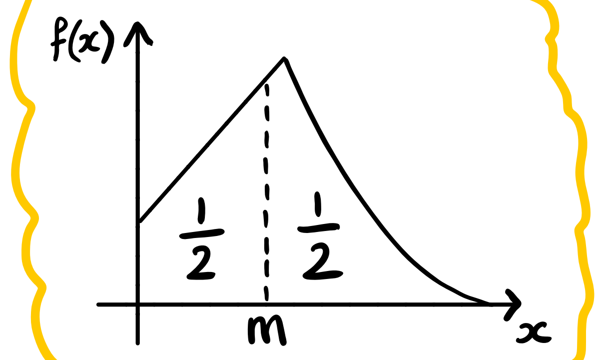
$$\mu = E(X) = \int_{-\infty}^{\infty} x f(x)\,dx$$

MODE = WHERE $f(x)$ HAS ITS MAX.

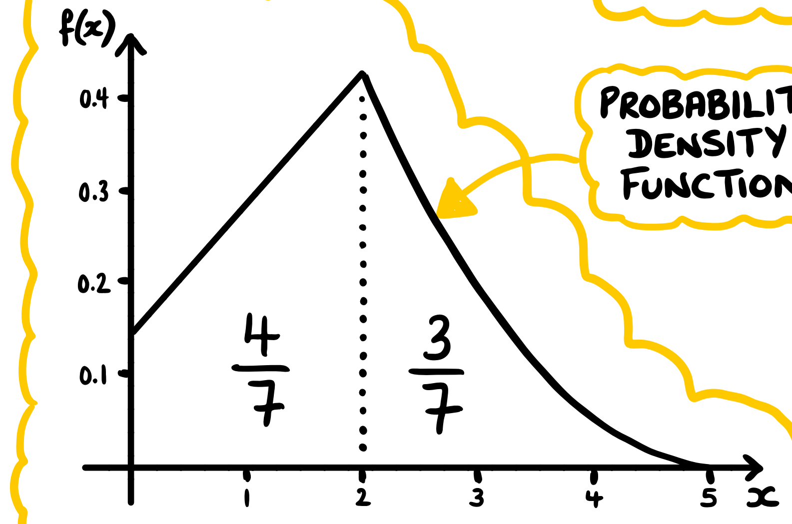
TOTAL AREA UNDER THE GRAPH IS 1 ⇒ $\int_{-\infty}^{\infty} f(x)\,dx = 1$

MEDIAN = m

$$\int_{-\infty}^{m} f(x)\,dx = \frac{1}{2}$$



PROBABILITY DENSITY FUNCTION



$$f(x) = \begin{cases} \frac{1}{7}(x+1), & 0 \le x \le 2 \\ \frac{1}{21}(x-5)^2, & 2 < x \le 5 \\ 0, & \text{OTHERWISE} \end{cases}$$

MODE = 2

$$\int_0^2 \frac{1}{7}(x+1)\,dx + \int_2^5 \frac{1}{21}(x-5)^2\,dx = \frac{4}{7} + \frac{3}{7} = 1$$

⇒ f IS A P.D.F.

$$\mu = E(X) = \int_0^2 x\cdot\frac{1}{7}(x+1)\,dx + \int_2^5 x\cdot\frac{1}{21}(x-5)^2\,dx = 0.667 + 1.179 = 1.845$$

$$E(X^2) = \int_0^2 x^2\cdot\frac{1}{7}(x+1)\,dx + \int_2^5 x^2\cdot\frac{1}{21}(x-5)^2\,dx = 0.952 + 3.386 = 4.338$$

$$Var(X) = E(X^2) - \mu^2 = 4.338 - 1.845^2 = 0.933$$

IF MEDIAN = m THEN

$$\int_0^m \frac{1}{7}(x+1) = \frac{1}{2}$$

GDC SOLVER ⇒ m = 1.828

DISCRETE

MODE = MOST LIKELY VALUE OF x

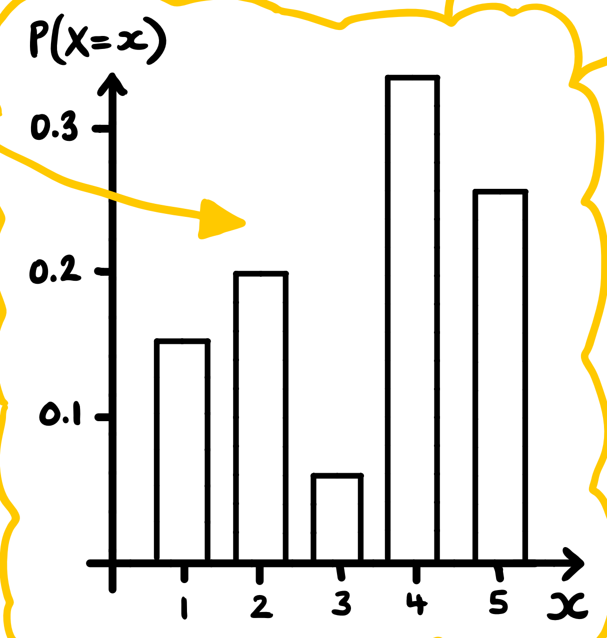
$$Var(X) = E(X^2) - [E(X)]^2 = \sum x^2 P(X=x) - \mu^2$$

$$\sum P(X=x) = 1$$

$$\mu = E(X) = \sum x P(X=x)$$

PROBABILITY DISTRIBUTION

x	1	2	3	4	5
$P(X=x)$	0.15	0.2	0.05	0.35	0.25



$$\mu = E(X) = (1\times0.15) + (2\times0.2) + \ldots + (5\times0.25) = 3.35$$

$$E(X^2) = (1^2\times0.15) + (2^2\times0.2) + \ldots + (5^2\times0.25) = 13.25$$

$$Var(X) = E(X^2) - \mu^2 = 13.25 - 3.35^2 = 2.028$$

LINEAR TRANSFORMATIONS OF X ⇒ aX+b

$$E(aX+b) = aE(X) + b$$

$$Var(aX+b) = a^2 Var(X)$$

e.g. $E(X) = 100$, $Var(X) = 20$ AND $Y = 3X+5$

⇒ $E(Y) = 3.100 + 5 = 305$, $Var(Y) = 3^2 \times 20 = 180$

Calculus

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DIFFERENTIAL EQUATIONS

FIRST ORDER EQUATIONS OF THE FORM...

VARIABLES SEPARABLE

$\frac{dy}{dx} = f(x)g(y)$

SEPARATE AND INTEGRATE

$\int \frac{1}{g(y)}\,dy = \int f(x)\,dx$

e.g. $\frac{dy}{dx} = xy - x = x(y-1)$

$\therefore \int \frac{1}{y-1}\,dy = \int x\,dx$

$\ln|y-1| = \frac{1}{2}x^2 + c$

$y - 1 = e^{\frac{1}{2}x^2} e^c$

$y - 1 = Ae^{\frac{1}{2}x^2}$

$y = Ae^{\frac{1}{2}x^2} + 1$

HOMOGENEOUS FORM

$\frac{dy}{dx} = f\left(\frac{y}{x}\right)$

SUBSTITUTE $y = vx$

e.g. $\frac{dy}{dx} = \frac{x+2y}{x} = 1 + 2\left(\frac{y}{x}\right)$

LET $y = vx \Rightarrow \frac{dy}{dx} = x\frac{dv}{dx} + v$

$\therefore x\frac{dv}{dx} + v = 1 + 2\left(\frac{vx}{x}\right)$

$x\frac{dv}{dx} = 1 + v$

$\int \frac{1}{1+v}\,dv = \int \frac{1}{x}\,dx$

$\ln|1+v| = \ln|x| + c$

$1 + \frac{y}{x} = Ax$

$y = Ax^2 - x$

INTEGRATING FACTOR

$\frac{dy}{dx} + P(x)y = Q(x)$

MULTIPLY THROUGH BY $I(x) = e^{\int P(x)\,dx}$

e.g. $\frac{dy}{dx} + 2y = e^x$

$\therefore I(x) = e^{\int 2\,dx} = e^{2x}$

$e^{2x}\frac{dy}{dx} + 2e^{2x}y = e^{3x}$

$\frac{d}{dx}(y \cdot e^{2x}) = e^{3x}$

$ye^{2x} = \int e^{3x}\,dx$

$ye^{2x} = \frac{1}{3}e^{3x} + c$

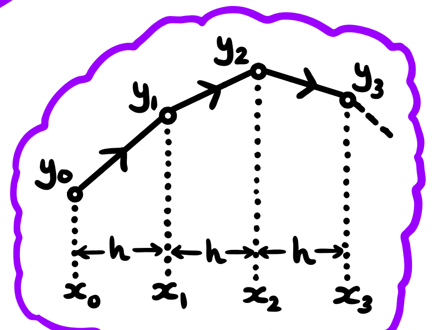
$y = \frac{e^x}{3} + \frac{c}{e^{2x}}$

NUMERICAL SOLUTIONS OF $\frac{dy}{dx} = f(x, y)$

EULER'S METHOD WITH CONSTANT STEP h

$y_{n+1} = y_n + hf(x_n, y_n)$

$x_{n+1} = x_n + h$



eg $\frac{dy}{dx} = x + y$

$x_0 = 0,\ y_0 = 1,\ h = 0.2$

n	x_n	y_n
0	0	= 1
1	0.2	= 1 + 0.2(0 + 1) = 1.2
2	0.4	= 1.2 + 0.2(0.2 + 1.2) = 1.48
3	0.6	= 1.48 + 0.2(0.4 + 1.48) = 1.856
4	0.8	= 1.856 + 0.2(0.6 + 1.856) = 2.3472
5	1	= 2.3472 + 0.2(0.8 + 2.3472) = 2.9766

$x_5 = 1,\ y_5 = 2.9766$

MACLAURIN SERIES

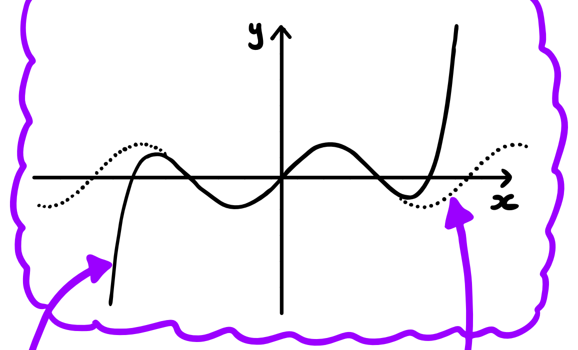
$f(x) = f(0) + f'(0)x + \frac{f''(0)}{2!}x^2 + \dots$

$f(0) = \cos(0) = 1$
$f'(0) = -\sin(0) = 0$
$f''(0) = -\cos(0) = -1$
$f'''(0) = \sin(0) = 0$
$f''''(0) = \cos(0) = 1$
etc...

$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \dots$

$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots$

$x \to x^2 \Rightarrow e^{x^2} = 1 + x^2 + \frac{x^4}{2!} + \frac{x^6}{3!} + \dots$



$y = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \frac{x^9}{9!}$

$y = \sin x$

FROM A DIFFERENTIAL EQUATION

$y' = x^2 + y^2,\ x = 0,\ y = 1$

DIFFERENTIATE IMPLICITLY

$\Rightarrow y'' = 2x + 2yy'$

$\Rightarrow y''' = 2 + 2yy'' + 2y'y'$

$y(0) = 1$

$y'(0) = 0^2 + 1^2 = 1$

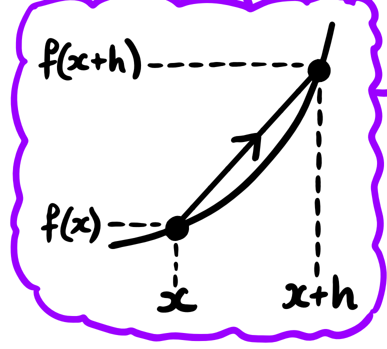
$y''(0) = 2.0 + 2.1.1 = 2$

$y'''(0) = 2 + 2.1.2 + 2.1.1 = 8$

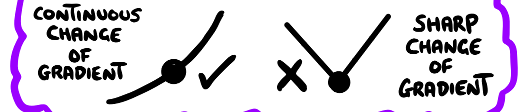
$y = 1 + 1.x + \frac{2.x^2}{2!} + \frac{8x^3}{3!} + \dots$

LIMITS

$f'(x) = \lim_{h \to 0}\left(\frac{f(x+h) - f(x)}{h}\right)$

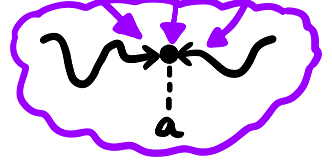


DIFFERENTIABLE AT $x = a$



CONTINUOUS AT $x = a$ IF

$\lim_{x \to a^-} f(x) = f(a) = \lim_{x \to a^+} f(x)$



L'HÔPITAL'S RULE

"$\frac{0}{0}$" or "$\frac{\infty}{\infty}$"

$\lim_{x \to a} \frac{f(x)}{g(x)} = \lim_{x \to a} \frac{f'(x)}{g'(x)}$

ONCE

$\lim_{x \to 1} \frac{x^3 + x^2 - 2}{3x^2 - 2x - 1} = \lim_{x \to 1} \frac{3x^2 + 2x}{6x - 2} = \frac{5}{4}$

TWICE

$\lim_{x \to \infty} \frac{x^2}{e^x} = \lim_{x \to \infty} \frac{2x}{e^x} = \lim_{x \to \infty} \frac{2}{e^x} = 0$

INTEGRATION BY SUBSTITUTION

$\int_{\text{HARD IN } x} dx \xrightarrow{\text{LET } u = f(x)} \int_{\text{EASY IN } u} du$

$\int x(2x+3)^5\,dx$

LET $u = 2x + 3$
$\frac{du}{dx} = 2,\ dx = \frac{1}{2}du$
$x = \frac{1}{2}(u - 3)$

$= \int \frac{1}{2}(u-3)u^5 \frac{1}{2}\,du$

$= \int \frac{1}{4}u^6 - \frac{3}{4}u^5\,du$

$= \frac{1}{28}u^7 - \frac{1}{8}u^6 + c$

$= \frac{(2x+3)^7}{28} - \frac{(2x+3)^6}{8} + c$

CHANGE LIMITS

$\int_0^{\ln 3} \frac{e^x}{e^{2x} + 9}\,dx$

$u = e^x,\ \frac{du}{dx} = e^x,\ du = e^x dx$
WHEN $x = 0 \Rightarrow u = 1$
$x = \ln 3 \Rightarrow u = 3$

$= \int_1^3 \frac{1}{u^2 + 9}\,du$

$= \left[\frac{1}{3}\arctan\frac{x}{3}\right]_1^3$

$= \frac{1}{3}\arctan 1 - \frac{1}{3}\arctan\frac{1}{3}$

$= \frac{\pi}{12} - \frac{1}{3}\arctan\frac{1}{3}$

TRIG SUBSTITUTIONS

IF YOU SEE	THEN USE
$\sqrt{a^2 - x^2}$	$\Rightarrow x = a\sin\theta$
$\sqrt{x^2 - a^2}$	$\Rightarrow x = a\sec\theta$
$\sqrt{a^2 + x^2}$	$\Rightarrow x = a\tan\theta$

INTEGRATION BY PARTS

$\int u\frac{dv}{dx} = uv - \int v\frac{du}{dx}\,dx$

MULTIPLY BY 1

$\int \ln x\,dx$
$= \int \ln x.1\,dx$
$= \ln x.x - \int x\frac{1}{x}\,dx$
$= x\ln x - x + c$

PARTS ONCE

$\int xe^x\,dx$
$= xe^x - \int e^x.1\,dx$
$= xe^x - e^x + c$

LOOP ROUND

$I = \int e^x \sin x\,dx$
$= e^x \cdot -\cos x + \int e^x \cos x\,dx$
$= -e^x\cos x + [e^x\sin x - \int e^x \sin x\,dx]$
$= -e^x\cos x + e^x\sin x - I$
$\therefore 2I = e^x\sin x - e^x\cos x$
$I = \frac{1}{2}(e^x\sin x - e^x\cos x) + c$

PARTS TWICE

$\int x^2\sin x\,dx$
$= x^2 \cdot -\cos x + \int 2x\cos x\,dx$
$= -x^2\cos x + [2x\sin x - \int 2\sin x\,dx]$
$= -x^2\cos x + 2x\sin x + 2\cos x + c$

MORE INTEGRALS

$\int \sec^2 x\,dx = \tan x + c$
$\int \sec x.\tan x\,dx = \sec x + c$
$\int \operatorname{cosec} x.\cot x\,dx = -\operatorname{cosec} x + c$
$\int \operatorname{cosec}^2 x\,dx = -\cot x + c$

$\int a^x\,dx = \frac{1}{\ln a}a^x + c$

$\int \frac{1}{\sqrt{1-x^2}}\,dx = \arcsin x + c$
$\int \frac{-1}{\sqrt{1-x^2}}\,dx = \arccos x + c$
$\int \frac{1}{1+x^2}\,dx = \arctan x + c$

MORE DERIVATIVES

$f(x) = \tan x \Rightarrow f'(x) = \sec^2 x$
$f(x) = \sec x \Rightarrow f'(x) = \sec x \tan x$
$f(x) = \operatorname{cosec} x \Rightarrow f'(x) = -\operatorname{cosec} x \cot x$
$f(x) = \cot x \Rightarrow f'(x) = -\operatorname{cosec}^2 x$

$f(x) = a^x$
$f'(x) = a^x \ln a$

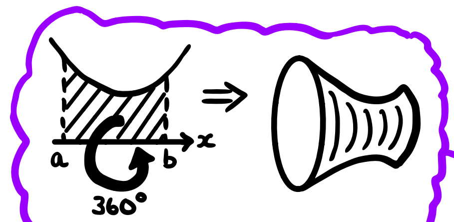
$f(x) = \log_a x$
$f'(x) = \frac{1}{x\ln a}$

$f(x) = \arcsin x$, $f'(x) = \frac{1}{\sqrt{1-x^2}}$

$f(x) = \arccos x$, $f'(x) = \frac{-1}{\sqrt{1-x^2}}$

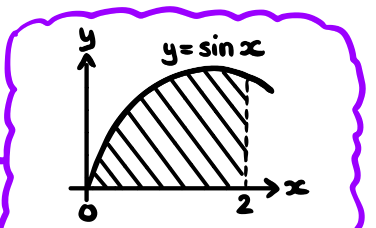
$f(x) = \arctan x$, $f'(x) = \frac{1}{1+x^2}$

VOLUME OF REVOLUTION



360° ABOUT THE x-AXIS

VOLUME $= \int_a^b \pi y^2\,dx$



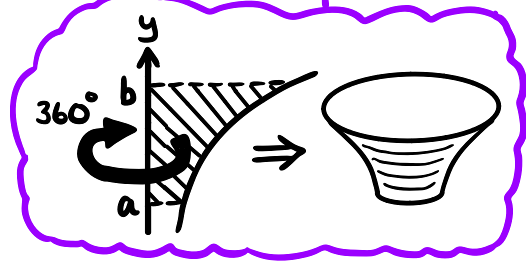
ROTATE THE SHADED AREA 360° ABOUT THE x-AXIS

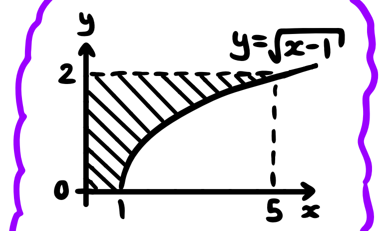
$\therefore V = \pi\int_0^2 \sin^2 x\,dx$

$= 3.74$ units^3

360° ABOUT THE y-AXIS

VOLUME $= \int_a^b \pi x^2\,dy$





ROTATE THE SHADED AREA 360° ABOUT THE y-AXIS

$y = \sqrt{x-1} \Rightarrow y^2 + 1 = x$

$\therefore V = \int_0^2 \pi(y^2+1)^2\,dy$

$= 43.14$ units^3

INTEGRATION WITH PARTIAL FRACTIONS

PARTIAL FRACTIONS

$\Rightarrow \frac{1}{x^2+3x+2} = \frac{1}{(x+1)(x+2)}$

$= \frac{1}{x+1} - \frac{1}{x+2}$

$\int \frac{1}{x^2+3x+2}\,dx = \int \frac{1}{x+1} - \frac{1}{x+2}\,dx$

$= \ln|x+1| - \ln|x+2| = \ln\left|\frac{x+1}{x+2}\right| + c$

IMPLICIT DIFFERENTIATION

CHAIN RULE

$y^5 \to 5y^4\frac{dy}{dx}$

FIND GRADIENT AT (a,b) ⇒ SUB. IN VALUES FIRST

PRODUCT RULE

$x^3y^2 \to x^3.2y\frac{dy}{dx} + 3x^2y^2$

$e^{2x+y} + 2xy^3 = 7$

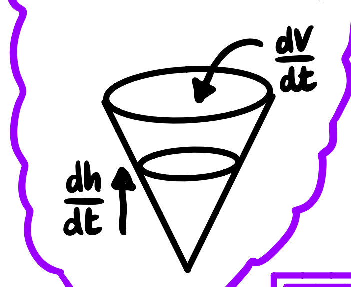
$e^{2x+y}\left(2 + \frac{dy}{dx}\right) + 2x.3y^2\frac{dy}{dx} + 2y^3 = 0$

$\frac{dy}{dx} = \frac{-2y^3 - e^{2x+y}}{e^{2x+y} + 6xy^2}$

RELATED RATES

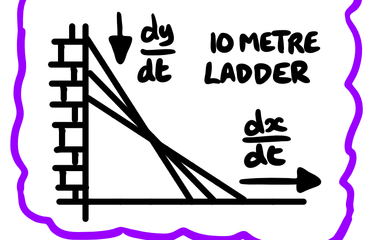
$V = \frac{1}{3}\pi r^2 h$

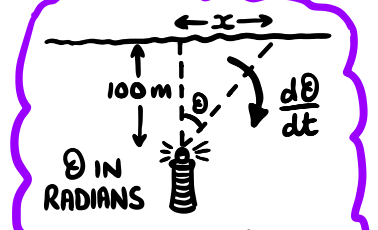
$\frac{dV}{dt} = \frac{1}{3}\pi\left(r^2\frac{dh}{dt} + 2r\frac{dr}{dt}h\right)$



$x^2 + y^2 = 100$

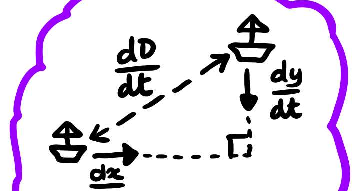
$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0$





$\tan\theta = \frac{x}{100}$

$\sec^2\theta\frac{d\theta}{dt} = \frac{1}{100}\frac{dx}{dt}$



$D^2 = x^2 + y^2$

$2D\frac{dD}{dt} = 2x\frac{dx}{dt} + 2y\frac{dy}{dt}$